Precalc Warm Up # 9-1

1. Find a polynomial of least degree and leading coefficient 1 with zeros: 6 and 1 + 2i. Write it in standard form.

2. Divide $8x^4 + 2x^3 + 3$ by $x^3 + 4x - 1$

3. Simplify: a) i^{422} b) $\sqrt{-9}\,\sqrt{-4}$

HW Questions: p. 226

In Exercises 1–26, find all the zeros of the function and write the polynomial as a product of linear factors.

3. $h(x) = x^2 - 4x + 1$

$(x-\ \ )(x-\ \ )$

$x - k$

zero

$0 = x^2 - 4x + 1$

$x^2 - 4x + \underline{4} + 1 - 4$

$0 = (x-2)^2 - 3$

7. $f(z) = z^2 - 2z + 2$

9. $g(x) = x^3 - 6x^2 + 13x - 10$

$\pm 1, \pm 2, \pm 5, \pm 10$

$g(x) = (x-2)(x^2 - 4x + 5)$

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 13 & -10 \\ & & 2 & -8 & 10 \\ \hline & 1 & -4 & 5 & 0 \end{array}$$

$(x-2)(x-\ \ )(x-\ \ )$

$0 = x^2 - 4x + \underline{4} + \underline{5 - 4}$

$0 = (x-2)^2 + 1$

$\pm\sqrt{-1} = \sqrt{(x-2)^2}$

$x - 2 = \pm i$

15. $f(x) = 16x^3 - 20x^2 - 4x + 15$ — zeros: $-\frac{3}{4}$, $1 \pm \frac{i}{2}$

$$\begin{array}{r|rrrr} -\frac{3}{4} & 16 & -20 & -4 & 15 \\ & & -12 & 24 & -15 \\ \hline & 16 & -32 & 20 & \end{array}$$

$(x+\frac{3}{4})(16x^2-32x+20)$

$4(x+\frac{3}{4})(4x^2-8x+5)$ ← use quadratic formula

21. $g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$

In Exercises 27–36, find a polynomial with integer coefficients that has the given zeros.

27. $1, 5i, -5i$

$f(x) = (x-1)(x-5i)(x+5i)$

$= (x-1)[x^2-(5i)^2]$

$= (x-1)(x^2+5)$

$f(x) = x^3 - x^2 + 5x - 5$

31. $i, -i, 6i, -6i$

$y = (x-i)(x+i)(x-6i)(x-(-6i))$

multiply it all out.

35) $\frac{3}{4}, -2, -\frac{1}{2}+i$, also: $-\frac{1}{2}-i$

$y = (x-\frac{3}{4})(x+2)(x-(-\frac{1}{2}+i))(x-(-\frac{1}{2}-i))$

$= \frac{1}{4}(4x-3)(x+2)[((x+\frac{1}{2})-i)((x+\frac{1}{2})+i)]$

$= \frac{1}{4}(4x-3)(x+2)[(x+\frac{1}{2})^2-i^2]$

$= \frac{1}{4}(4x-3)(x+2)(x^2+x+\frac{1}{4}+1)$

$= \frac{1}{4}(4x-3)(x+2)(x^2+x+\frac{5}{4}) \rightarrow$ factor out $\frac{1}{4}$: $\frac{1}{4}(4x^2+4x+5)$

multiply this $\frac{1}{4}$ by the one in front.

$= \frac{1}{16}(4x-3)(x+2)(4x^2+4x+5)$

Now you can multiply the 3 ( )'s together :)

In Exercises 37–40, write the polynomial (a) as the product of factors that are irreducible over the *rationals*, (b) as the product of linear and quadratic factors that are irreducible over the *reals*, and (c) in completely factored form.

a) No $\sqrt{\ }$ or i

b) $\sqrt{\ }$ ok, but no i

c) all linear factors.

39. $f(x) = x^4 - 4x^3 + 5x^2 - 2x - 6$
[*Hint:* One factor is $x^2 - 2x - 2$.]

so $\div$

$$x^2 - 2x - 2 \overline{)\, x^4 - 4x^3 + 5x^2 - 2x - 6}$$

to find the other quadratic factor.

In Exercises 41–50, use the given zero of f to find all the zeros of f.

43. $f(x) = 2x^4 - x^3 + 7x^2 - 4x - 4, \quad r = 2i \rightarrow$ so also $-2i$

47. $f(x) = x^4 + 3x^3 - 5x^2 - 21x + 22, \quad r = (-3 + \sqrt{2}i)$ $\quad (-3 - \sqrt{2}i)$

$(a-b)(a+b)$

$a^2 - b^2$

$$(x - (-3 + \sqrt{2}i))(x - (-3 - \sqrt{2}i))$$

$$((x+3) - (\sqrt{2}i))((x+3) + (\sqrt{2}i))$$

$$(x+3)^2 - (\sqrt{2}i)^2$$

$$x^2 + 6x + 9 - 2i^2$$

$$(x^2 + 6x + 11)$$

$$\begin{array}{r} x^2 - 3x + 2 \\ x^2 + 6x + 11 \overline{)\, x^4 + 3x^3 - 5x^2 - 21x + 22} \\ -(x^4 + 6x^3 + 11x^2) \\ \hline -3x^3 - 16x^2 - 21x + 22 \\ -(-3x^3 - 18x^2 - 33x) \\ \hline 2x^2 + 12x + 22 \\ 2x^2 + 12x + 22 \\ \hline 0 \end{array}$$

zeros: $1, 2$
$-3 + \sqrt{2}i$; $-3 - \sqrt{2}i$

$$(x^2 + 6x + 11)(x^2 - 3x + 2)$$

$$(x - 2)(x - 1)$$

51) $\pm\sqrt{6}\,i$

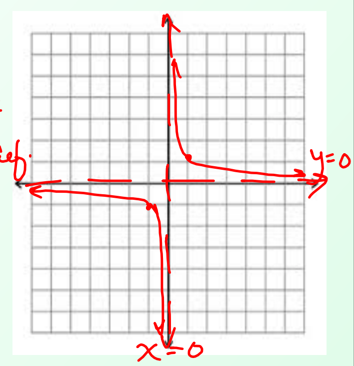
$f(x) = (x - \sqrt{6}\,i)(x - (-\sqrt{6}\,i))$

Graph $f(x) = \dfrac{1}{x}$ by picking points

Vertical Asymptote? $x = 0$

Horizontal Asymptote? $y = 0$

x	y
-2	-½
-1	-1
0	undef.
1	1
2	½



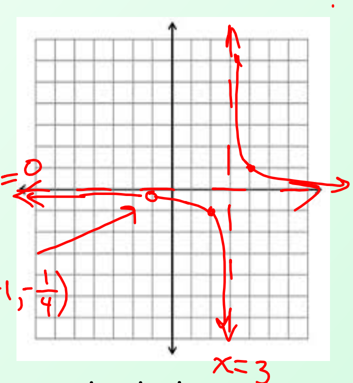
Graph. (Hint: factor first!)

$$t(x) = \frac{x+1}{x^2 - 2x - 3}$$

$(x-3)(x+1)$

x	y
4	1
2	-1

hole at $(-1, -\frac{1}{4})$



Vertical Asymptotes? $x = 3$

Horizontal Asymptotes? $y = 0$

Holes? at $x = -1$

Graph this with your grapher. Do you see the holes?

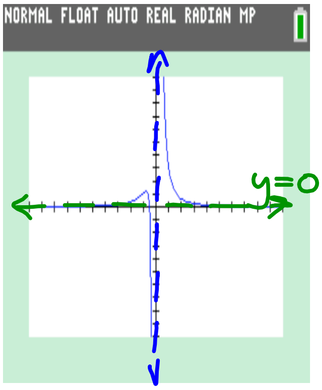
$$s(x) = \frac{2x+1}{x^3}$$

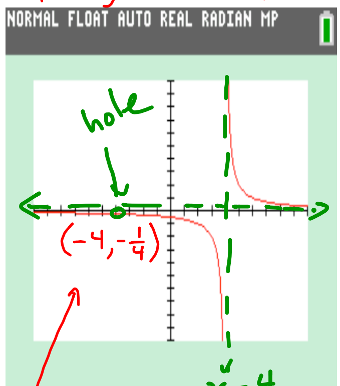
vert @ $x=0$

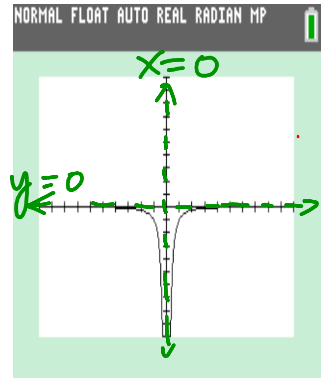
$$j(x) = \frac{\overset{2(x+4)}{2x+8}}{\underset{(x+4)(x-4)}{x^2-16}}$$

$$e(x) = \frac{-1}{(x)^2}$$

use $y = \frac{2}{x-4}$ to find the y-value of the hole!







$$y = \frac{2}{-4-4}$$
$$= \frac{2}{-8}$$

Vertical asymptotes can never be crossed, because they are zeros of the denominator and are not allowed. Horizontal asymptotes effect END BEHAVIOR and can be crossed!

All five of the previous examples had the x axis (y=0) for the horizontal assymptote. This makes sense if we use a little Calculus. It is clear that as x approaches infinity or negative infinity, the following functions approach 0, so y = 0 is a horizontal asymptote.

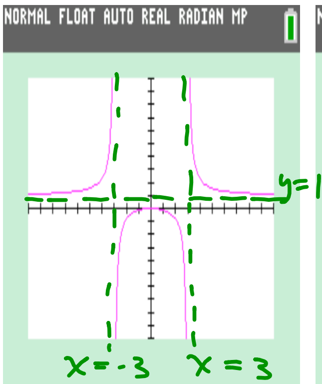
$\frac{\text{Small \#}}{\text{Huge \#}}$

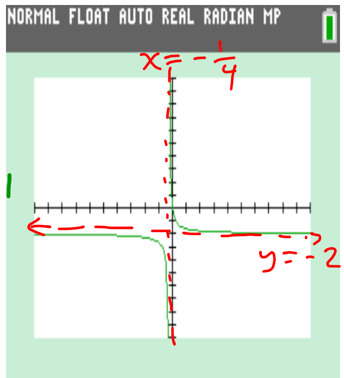
$$s(x) = \frac{2x+1}{x^3} \qquad j(x) = \frac{2x+8}{x^2-16} \qquad e(x) = \frac{-1}{x^2}$$

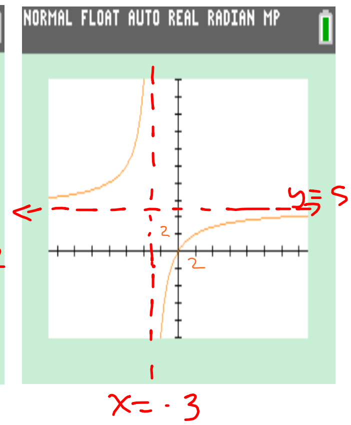
All of the functions that we just graphed were examples of rational functions where the degree of the numerator was LESS than that of the denominator. Whenever we have this type, the horizontal asymptote will always be the x axis, y = 0.

$$s(x)=\frac{1x^2}{1x^2-9} \qquad j(x)=\frac{-8x}{4x+1} \qquad f(x)=\frac{5x}{1x+3}$$

$$y=\frac{-8}{4}=-2$$







When degree in numerator and denominator are the same, you divide leading coefficients. We can see this algebraically.

$$y=\frac{LC}{LC}$$

$$f(x)=\frac{5x}{x+3}$$

$y=5$

What is the horizontal asymptote? Try to algebraically show that y ≠ 5

$$5 \stackrel{?}{=} \frac{5x}{x+3}$$

$$5x+15=5x$$
$$-5x \qquad\qquad -5x$$

$$15\neq 0$$

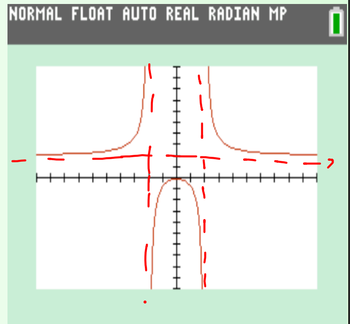
What is the horizontal asymptote?

$y = \frac{LC}{LC} = \frac{6}{3} = 2$

$$g(x) = \frac{6x^2+1}{3x^2-12}$$

$3(x^2-4)$

Vert :



What are the vertical asymptotes? Are there any holes? Graph this function without your grapher.

No holes. No factors that can cancel.

the verticals divide the graph into 3 regions.
Pick points in all 3 regions:

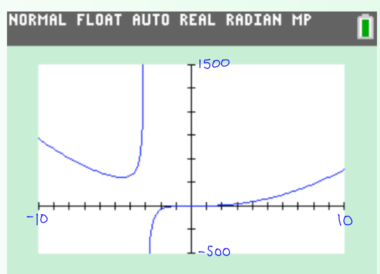
x	y
-3	
-1	
0	
1	
3	

When the degree of the numerator is MORE than that of the denominator, there are no horizontal asymptotes.

Vert: $x = -3$

Graph (with grapher) $f(x) = \frac{5x^3}{x+3}$ to see that this is true.

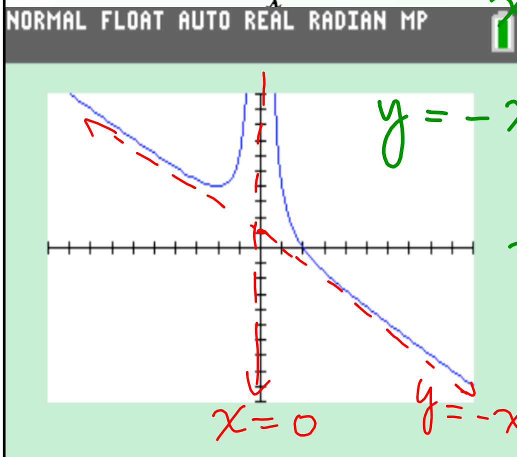
window
$x \to (-10, 10)$
$y \to (-500, 1500)$



If the degree in numerator is exactly one higher than that of the denominator, then no horizontal asymptote, but there is a SLANTED ASYMPTOTE!

To understand slanted asymptote, we use a little Calculus! Divide the numerator by the denominator, and imagine what happens when x gets very large or very small. We are taking the limit as $x \to \infty$ or $-\infty$

$$f(x) = \frac{-x^3 + x^2 + 4}{x^2} = \frac{-x^3}{x^2} + \frac{x^2}{x^2} + \frac{4}{x^2} \to 0$$



$y = -x + 1$

$$\begin{array}{r} -x + 1 \\ x^2 \overline{) -x^3 + x^2 + 4} \\ -(-x^3) \quad\quad\quad \\ \hline x^2 + 4 \\ -(x^2) \quad \\ \hline 4 \end{array}$$

ignore

HW PC Book:

p. 237 #1-8, 11-59 □

Group event this Thursday:
Graph rational functions by hand, without a grapher.